Sample Paper – 2014
Class – XII
Subject – Mathematics
CODE: 87-13SM-P1

TIME: 03 HOURS **MAX. MARKS: 100**

Instructions:

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of 01 mark each, Section B comprises of 12 questions of 04 marks each and section C comprises of 07 questions of 06 marks each.
3. Use of calculators is not permitted.
4. Students **MUST NOT** take back the question paper with them.

SECTION A

1. Write the equation of plane passing through (1, −2, 3) and perpendicular to line $\frac{x+1}{1} = \frac{y}{3} = \frac{z-2}{-2}$ in Cartesian form.

2. If $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$, find the value of y.

3. Prove that: $\tan^{-1}(-1)$ (−1) + $\cos^{-1}(-1)$ $\left(-\frac{1}{\sqrt{2}}\right) = \frac{\pi}{2}$.

4. Find the projection of vector **i** – 2**j** + **k** on the vector 4**i** – 4**j** + 7**k**.

5. If $|\vec{a}| = 5$, $|\vec{b}|$ = 13, $|\vec{a} \times \vec{b}|$ = 25 then find $\vec{a} . \vec{b}$.

6. If A = $\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ then find k if |2A| = k |A |

7. If * be the binary operation on Z_0 such that a*b = $a^2 - b^2$ + ab + 4, then find (2*3)*4

8. Find the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 6x & 9x & 12x \end{vmatrix}$.

9. Evaluate: $\int (x^a + a^x + e^x . a^x + \sin a)$ dx.

10. Evaluate: $\int_{-1}^{1}[x]$ dx, where [x] defined greatest integer function.

SECTION B

11. Prove that the image of the point (3,–2, 1) in the plane 3x – y + 4z = 2 lies on the plane, x + y + z + 4 = 0.

12. Solve the differential equation $(x^2 + xy)\,dy = (x^2 + y^2)\,dx$.

13. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a\left(x^3-y^3\right)$ prove that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$.

14. Let A = N × N and * be the binary operation on A defined by (a, b) * (c, d) = (a + c , b + d) show that * is commutative and associative. Find identity element for * on A, if any.

15. Prove that: $\tan\left(\frac{\frac{\pi}{4}+\frac{1}{2}\cos^{-1} a}{b}\right)+\tan\left(\frac{\frac{\pi}{4}-\frac{1}{2}\cos^{-1} a}{b}\right)=\frac{2b}{a}$.

16. Prove that $\begin{vmatrix}(b+c)^2 & ba & ac\\ ba & (a+c)^2 & bc\\ ac & bc & (b+a)^2\end{vmatrix}=2abc(a+b+c)^3$.

17. Evaluate: $\int\frac{1}{\sin(x-a)\cos(x-b)}$ dx.

18. Draw the graph of greatest integer function and Prove that it is discontinuous function at all integral point and continuous at integral points.

19. Three cards are drawn from a pack of 52 playing cards. Find the probability distribution of the number of aces.

20. Show that the equation of normal at any point on the curve

$x = 3\cos\theta - \cos^3\theta$, $y = 3\sin\theta - \sin^3\theta$ is $4(y\cos^3\theta - x\sin^3\theta) = 3\sin 4\theta$.

21. Solve the differential equation, $(1+ y + x^2y)dx + (x+ x^3)dy = 0$ where y = 0 when x = 1.

22. Prove that in a Δ ABC, $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$, where a, b, c represent the magnitude of the sides opposite to vertices A, B, C respectively.

SECTION C

23. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that the area of triangle is maximum value when $\theta = \frac{\pi}{6}$.

24. A dealer in rural area wishes to purchase a number of sewing machines. He has only Rs.5760.00 to invest and has space for at most 20 items. An electronic sewing machine costs him Rs.360.00 and a manually operated sewing machine Rs.240.00. He can sell an Electronic Sewing Machine at a profit of Rs.22.00 and a manually operated sewing machine at a profit of Rs.18.00. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Make it as a linear programming problem and then, solve it graphically. Keeping the rural background in mind justify the 'values' to be promoted for the selection of the manually operated machine.

25. Find the area bounded by the region $\left[(x,y): \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1 \le \frac{x}{a} + \frac{y}{b}\right]$

OR

Using integration, find the area of the region {(x, y); |x −1| ≤ y ≤ $\sqrt{5 - x^2}$ }

26. Using Matrices, Solve the following system of equation 2x – 3y + 5z = 11, 3x + 2y – 4z = −5, x + y – 2z = −3.

27. Evaluate: $\int_{-1}^{\frac{3}{2}} |x \sin \pi x| \, dx$

OR

Prove that $\int_0^a f(x)dx = \int_0^a f(a - x)\, dx$. Hence evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \tan x}$

28. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. Besides 'TRUTHFUL' name the life skills which we should acquire?

29. Find the distance of the point (−2, 3, −4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

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